**EC 1310-** Control System

**ROOT LOCUS PLOT**

**Root Locus**

In control systems theory, root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly a gain within a feedback system. This is a technique used as a stability criterion in the field of classical control theory developed by Walter R. Evans which can determine stability of the system. The root locus plots the poles of the closed loop transfer function in the complex s-plane as a function of a gain parameter

**Root Locus Plot**

This is also known as root locus technique in control system and is used for determining the stability of the given system. Now in order to determine the stability of the system using the root locus technique we find the range of values of K for which the complete performance of the system will be satisfactory and the operation is stable.  
Now there are some results that one should remember in order to plot the root locus. These results are written below:

1. Region where root locus exists : After plotting all the poles and zeros on the plane, we can easily find out the region of existence of the root locus by using one simple rule which is written below,  
   Only that segment will be considered in making root locus if the total number of poles and zeros at the right hand side of the segment is odd.
2. How to calculate the number of separate root loci ? : A number of separate root loci are equal to the total number of roots if number of roots are greater than the number of poles otherwise number of separate root loci is equal to the total number of poles if number of roots are greater than the number of zeros.

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All root locus rules can be directly traced to the characteristic equation, 1+*L*(*s*)=0. If we assume that the loop transfer function can be written as *L*(*s*)=*KL*0(*s*), where *K* is a positive gain, then we can write the magnitude condition and the angle condition as

|*L*(*s*)|=|*KL*0(*s*)|=1

\begin{displaymath}\angle L(s) = \angle L_0(s) = -180^\circ. \end{displaymath}

We assume that the loop transfer function has *P* open loop poles and *Z* open loop zeros, and that there are at least as many poles as zeros ($P\geq Z$).

**Rule 1**

The number of branches, which are the paths of the closed-loop poles, is equal to the number of open-loop poles, *P*.

**Rule 2**

The branches start at the open-loop poles and end at the open-loop zeros. In addition to the *Z* explicit open-loop zeros in the transfer function, there are *P*-*Z* open-loop zeros at infinity.

**Rule 3**

Branches of the root locus lie on the real axis to the left of an odd number of poles and zeros. Complex-conjugate pairs of poles and zeros are not counted, since they contribute no net angle to the real axis.

**Rule 4**

If a branch on the real axis lies between a pair of poles, the root locus must break away from the real axis somewhere between the poles. Similarly, if a branch on the real axis lies between a pair of zeros, there must be an entry point between that pair of zeros.

**Rule 5**

As *K* gets very large, *P*-*Z* branches go to infinity. These branches approach asymptotes at angles to the real axis of

\begin{displaymath}\alpha_n = \frac{(2n+1)180^\circ}{P-Z}\end{displaymath}

where $n=0\ldots(P-Z-1)$ and the centroid of these asymptotes is on the real axis at

\begin{displaymath}\sigma_a = \frac{\sum p_i - \sum z_j}{P-Z}. \end{displaymath}

**Rule 6**

The departure angles   of the branches from an *m*th-order pole on the real axis are

\begin{displaymath}\delta_n = \frac{(2n+1)180^\circ}{m}\end{displaymath}

if the *m*th-order pole is to the left of a even number of poles and zeros. If the *m*th-order pole is to the left of a odd number of poles and zeros, then the departure angles are

\begin{displaymath}\delta_n = \frac{2n180^\circ}{m}.\end{displaymath}

**Rule 7**

If there are two or more excess poles than zeros ( $P-Z \geq 2$), then for any gain *K*, the sum of the real parts of the closed-loop poles (or the average distance from the $j\omega$-axis) is constant[3](http://www.mit.edu/people/klund/weblatex/footnode.html" \l "foot2284).

**Rule 8**

Ignore remote poles and zeros when considering the root locus near the origin of the *s*-plane, and combine the poles and zeros near the origin when considering the root locus for remote poles and zeros.

**Rule 9**

The departure angle from a complex-conjugate pole can be found by considering the angle condition on a small circle around the pole. The result is found by summing all the angles from open-loop zeros and subtracting all the angles from all other poles

\begin{displaymath}\delta_P = 180^\circ+\sum\angle z_i-\sum\angle\ p_j \end{displaymath}

The approach angle to a complex-conjugate zero follows similarly

\begin{displaymath}\delta_Z = 180^\circ-\sum\angle z_i+\sum\angle\ p_j \end{displaymath}

This sum only needs to be calculated once for each complex pair, since the root-locus diagram is symmetric above and below the real axis.

**Rule 10**

The break-away (entry) points from (to) the real axis between a pair of poles (zeros) can be found either by geometric construction[4](http://www.mit.edu/people/klund/weblatex/footnode.html" \l "foot2293) or by finding the local maxima (minima) of the gain function $K(\sigma)$, solving 

\begin{displaymath}\frac{\partial}{\partial\sigma}K(\sigma) =
\frac{\partial}{\partial\sigma} \left\vert\frac1{L(\sigma)}\right\vert = 0. \end{displaymath}

Fortunately, this level of accuracy is rarely necessary.